Reeding Notes :

Discussion, Comments and Answers

P. Evesque: Reading notes on two interesting works :

1) On "La transition de Jamming dans un milieu granulaire bidimensionnel: Statique et dynamique d'un système athermique modèle" by Frédéric Lechenault ; PhD thesis, Univ Paris Sud XI (2007) ; preprint thèse version Nov 2007 arXiv:cond-mat/0502504v1 ;

& F Lechenault, F da Cruz, O Dauchot and E Bertin' Free volume distributions and compactivity measurement in a bidimensional granular packing, J. Stat. Mech. (2006) P07009

2) On « on the rigidity of amorphous solids » by M. Wyart, First part of PhD Thesis Nov 2005, Ecole polytechnique, Palaiseau, France).

These two works are quite interesting. They are both dedicated to the quasi-statics of granular media.

1) on Lechenault approach:

In particular the work of Lechenault investigates the distribution of voids in a 2d sample made of two kinds of cylinders having different diameters. This is to test the applicability of the compactivity concept, and of some rule on entropy. Unfortunately, it finds that the distribution of void fluctuations does not obey the central limit theorem, which seems then to demonstrate the inapplicability of these concepts. This is just this point that I would like to discuss and the pertinence of the extrapolation of these results to 3d systems with a single kind of spheres:

In fact it is well known in soil mechanics that 2-d systems with a single kind of cylinders display anomalous behaviour due to too large "hexagonal" ordering. This ordering result from a local order which is due to a maximum density which organises itself further and propagate to infinity. It increases collective effects due long range order and increase anomalously the effect of dilatancy mechanism. Hence it is often used in soil mechanics what is called analogous Schnebelli systems which consists to use a medium made of a mixing of cylinders with two different sizes.

But it is known from long that such Schnebelli system does not lead to amplitude of fluctuations similar to those ones in 3d [1, 2]. So the 2d bi-disperse and 3d mono-disperse systems are not strictly equivalent from the mechanics point of view.

The second point to consider is that binary systems of rods, of spheres or of cylinders generate spontaneously size-segregation. (This is a well established fact, which occurs within one round of a rotating drum for instance). So it is normal to suppose that segregation may exist also in the system studied by Lechenault. But who says segregation, tells non random distribution, hence tells some difference from the normal Gaussian regression of fluctuations. So the interesting result obtained by Lechenault, may simply **evidence some segregation** problem.

Soil mechanics specialists have never studied the 2d rheological law of bi-disperse systems in view of studying segregation, and quasi-statics may not be the easiest way to study this trend, since relative motion of grains is quite small during the test because the deformation range remains small; hence the increase/change of segregation shall remain small. However, as soon as deformation range becomes large, segregation develops (silo emptying is a good example of quasi-statics flow with large deformation near the walls).

So a question arises: Does 2-d Lechenault result applies also in 3d? I do not believe so for the following reasons: as a matter of fact I believe that the pressure dependence of the void ratio [3] (or of the porosity) which is found in soil mechanics, i.e. 3d soil mechanics, is an evidence of the applicability of the statistical concepts to the 3d quasi-static mechanics of granular media; in particular, I have interpreted the experimental pressure dependence of the void index as such an evidence in 1999 already [3]. And the reasoning I used there follows an idea developed by Boutreux and de Gennes [4] to explain the density dependence of a vibrated packing upon vibration which was referring to the Edwards compactivity concept. But further 3d studies in the spirit of Lechenault one would be quite nice to perform to demonstrate this fact. This may need to use X-ray or NMR Imaging.

Also, the force distribution obeys classic statistics directly issued from maximum entropy principles, (see refs [5, 6] to know my point of view; but few other authors have developed similar ideas).

On the other hand, as segregation develops in 2-d bi-disperse case, does it mean that one shall predict also that the regression of fluctuations of stress with sample size will not follow exactly the central limit theorem in a 2-d mixture of grains? This shall be confirmed or infirmed.

At last it shall be emphasized also that another artefact can also come from the role plays by boundaries, by introducing a specific length scale. But this will occur also in 3d samples.

Hence the compactivity concept cannot apply as simply as it is proposed in Lechenault study when segregation occurs. In this peculiar case, it would be better to use a theory similar to the one proposed by de Larrard [7] in his thesis.

Conclusion: These remarks mean that it may be merely impossible to try and test accurately the ideas of compactivity and entropy of the void distribution on 2d granular systems. It would be better to try testing it directly in 3-d mono-disperse systems.

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2) « on the rigidity of amorphous solids » by M. Wyart

This work starts from the idea of soft modes. This is a powerful idea. The determination of the number of soft modes as a function of the system size in an isostatic system is then considered and applied with efficiency. This leads to the flat density of states when the mode frequency tends to 0. (If I have well understood, the prediction of this constancy of the soft mode density with frequency at small frequency comes from isostaticity hypothesis: since force shall propagate through the sample in such a system, and since deformation shall be possible cutting one contact, the number of soft modes scale as L^{d-1} corresponding to the halve of the contacts in the total surface.

However, I have not fully understood the method of decimation: is there a one to one correspondence between the number of soft modes and the half number of grains at the boundary? Is this number linked to the existence of some deformation plane which develops through the sample (and in this case is some kind of localisation similar to what is obtained in experiments). What are the theorems about this questions, if they are?

Remark 1: I just realize that few examples of soft modes can be found probably in [8]; but they are called deformation modes there.

Remark 2: In fact there might be some important differences between non frictional grains and frictional grains (see [8-10]); so what is the true behaviour with friction grains?

- Roux and co-workers have studied the two cases for instance [10]; the simulations reported in [10] show that the behaviour is quite anomalous in the first case while one can define a stress-strain law in the second case [9]. This is due to dissipation which should not play any role in the first case (but which does because of convergence problem).
- Also it is demonstrated in [8] that for a given configuration of the contacts and of the forces, the deformation modes define a half space vector when friction is introduced, while they define a complete space vector without solid friction. Does this modify the distribution of soft modes? Probably this is not important if one considers the stable limit system, which is able to transport infinitely small vibrations. But the true soft modes are vibration modes, with amplitude which is alternatively positive and negative, and the true state is stable only in one direction. Hence theoretical approach as the one developed in [11-12] shall be applied instead of the classic approach without hysteresis.

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Questions to both authors:

I have now two questions to both authors: Can they agree with what I have developed in these lines and in refs [4, 8, 9] (for me I believe the answer is yes, mainly). And if there are points of disagreement, which are they?

I can ask also an other question which I first formulated in [9]: one can consider in an isostatic system that each ingoing force on a side of the system shall correspond to an outgoing one on the other side of the sample (due to the number of unknowns), so that stress propagates through. But this implies also that such a sample obeys some plastic flow rule that links stress components together. In turn it shall impose stress to propagate along lines [13-15]. So as one knows how to generate a system in a perfectly plastic state, one should be able also to observe stress propagation along lines. Why doesn't one see such an effect often? Is it because the plastic rule is broken as soon as the local forcing is stressed?

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